Accounting Methods and Managerial Discretion: The Case of Dollar-Value LIFO

Timothy L. Shaftel
University of Kansas, Lawrence, KANSAS, USA

This paper focuses on inconsistencies in cost-flow behavior associated with the use of dollar-value LIFO. These inconsistencies can lead to confusing inventory valuations, potentially misleading accounting reports, incorrect purchasing decisions, and unanticipated tax results. Understanding these effects will enhance the use of dollar-value LIFO as a management tool and will improve the interpretation of financial accounting information. With dollar-value LIFO for multiple items, much of the conventional wisdom about LIFO vanishes. The relationships among inventory purchases, inventory liquidations, and gross margin are derived.

INTRODUCTION

Dollar-value LIFO allows inventory items to be pooled into large, not necessarily economically related, classes. The total value of inventory is then based on the present cost of the individual items in the pool and on an index generated each year for the entire pool. Dollar-value LIFO allows reductions in the stock of one item to be offset by increases in other items in a manner that permits better management of overall inventory levels. As a consequence, multi-item pools are common. Reeve and Stanga (1987) report that for 206 firms using LIFO, 95% relied on the dollar-value method. To date, academic research has mostly overlooked the special properties of this important inventory technique. In this paper the relationships among inventory purchases, inventory liquidations, and gross margin are derived for dollar-value LIFO. These relationships will be used to show that the potential impact of purchases on reported net income, taxes, and cash flow can be much greater with multi-item pools than with the single-item model. In addition, these relationships will be used to demonstrate that several long-held beliefs regarding LIFO inventory methods are not generally valid for dollar-value LIFO.

Previous empirical and analytical research has focused on inventory-related decisions made by managers. One area that has been studied empirically is the choice of cost-flow assumptions, including tax incentives and other hypothesized 'non-tax' reasons for making these choices. Dopuch and Pincus [1988] review previous work in this area and provide additional evidence on tax and non-tax explanations for a manager's behavior. Many of the empirical studies require conversion from LIFO to FIFO values, or vice versa, using an assumed external index and the implied assumption of single-item inventories. Analytical research, which focuses on optimal inventory decision making, has been carried out by Cohen and Halperin (1980), Biddle and Martin (1985, 1986), and Bowen and Pfeiffer (1989). These papers all attempt to integrate aspects of taxes, inflation, carrying costs, and timing into the decision process when inventory is of a single-item type. One exception to this was presented in Cron and Hayes (1989). In that paper, the authors studied the concept of multiple inventory pools while ignoring the impact of having more than one item within each pool. The analysis further assumed that the inventory quantities of each item would be increasing in every period. Under this limiting assumption, decision rules were devised for selecting the number of inventory pools.

Estimates of the potential tax savings associated with LIFO procedures can be found in several papers, including Biddle (1980) and Biddle and Lindahl (1982). In the latter, the authors found that

0143–6570/92/020119–15$07.50

311 firms adopting LIFO saved an average of $10 million each in the first year alone. The idea that potential savings exist is not a new one. Butters (1949) pointed to the existence of large tax incentives from using LIFO. Further, he stated that if older layers are liquidated, then 'the difference between LIFO costs and current costs will be brought into income'. Recent research expands on this observation. In Davis *et al.* [1984], for instance, the authors state that 'A LIFO firm has definite tax disincentive of liquidation only if current costs exceed historical costs for all potential liquidations'. Similar statements can be found elsewhere in the literature. As stated earlier, these statements are not true in general when the implicit assumption of single-item inventories is removed.

Dollar-value LIFO is accepted by the IRS, the SEC and GAAP for inventory valuation purposes. Various rules for its use have been promulgated by the IRS for tax purposes, but few others exist. The SEC does provide some guidance for decision makers in ARS 293 with examples of inappropriate applications of LIFO. In addition, the AICPA has clarified many procedural questions in an Issues Paper released in 1984. Because policy in this area is extremely complex, much of the discussion and suggestions in that release are general in nature. The IRS interpretations of the LIFO conformity rule are often cited as the reason for the lack of specific financial reporting regulations related to inventory values. The conformity rule states that companies using LIFO for tax purposes must also use it for external financial reporting. This requirement has been greatly relaxed in recent years. In fact the IRS specifically allows costing methods such as the type of index used, the index sample size, and the configuration of inventory pools for financial reporting purposes to be different from those used for tax purposes (Treasury Regulations, Section 1.472-2(e)(8)). Because of this relaxation and since the IRS does not generally address financial accounting issues, managers are left with a great deal of control over reported inventory figures.

The mathematics of dollar-value LIFO and the relationship between inventory and gross margin is described in the next section. In addition, the usually accepted effect of inventory on net income is derived. The third section of the paper contains discussions of internally developed inventory cost indices and their impact on gross margin and inventory quantities. The effect of incorrectly specified external or internal cost indices is also described in this section. Sample calculations and conclusions are presented in the final two sections.

DETERMINING DOLLAR-VALUE INVENTORY

For any pool, the dollar-value LIFO procedure begins by valuing ending inventory for the period at the year's cost. Companies can elect to use the beginning, ending or average cost for the year for each item type. This value is then adjusted to a base-period value by dividing by an inventory cost index that reflects changes due to inflation. The resulting adjusted inventory valuation is compared to the same value for the preceding year. If the new value is greater than that of the preceding year the difference is added as a new inventory layer. If the new value is less than the preceding year's, layers are removed, beginning with the most recent. Each layer is then multiplied by its appropriate cost index and the results added to determine the new dollar-value inventory amount. (See Chasteen *et al.*, 1987, or Kieso and Weygandt, 1986, for a description of this procedure.) For the mathematical formulation:

I = the set of inventory items $I=\{1, \ldots n\}$,
c_{ij} = the cost of item i for year j.
q_{ij} = the amount of item i in inventory at the end of year j,
Q_j = the value of existing inventory at the end of year j, in terms of year j's costs (i.e. the current replacement value),
B_j = the value of existing inventory at the end of year j, in terms of year j's costs adjusted to the base year (referred to as the adjusted inventory),
f_j = the inventory cost index for year j.

The base-year values are represented when $j=0$. Then:

$$Q_j=\sum_I c_{ij}q_{ij} \quad (1)$$

$$B_j=\sum_I c_{ij}q_{ij}/f_j \quad (2)$$

Inventory layers are added over time. As stated earlier, a single inventory layer is added in year j if the adjusted inventory for year j, B_j, exceeds the adjusted inventory for the previous year, B_{j-1}. If B_j is less than B_{j-1}, inventory is removed from the most recent layer until it reaches zero. Inventory is then removed from the next most recent layer, and so on, until the sum of the remaining layers equals the value of the adjusted inventory for the year. For ease of presentation, the formulation will require a layer to be associated with each year, but the value may be zero. In addition, the index specifying an inventory layer will be retained even when that layer is 'eliminated' by its subsequent reduction to zero. Adjusted inventory layers are created and modified over time. Define L_{kj} to be the adjusted value in year j of the inventory layer created in year k ($k=0$ represents inventory in the base year). Since all future years' layers are zero, define $L_{kj}=0$ for $k>j$. Thus, by construction of the inventory layers for each year:

$$B_j=\sum_{k=0}^{j} L_{kj} \tag{3}$$

Assume that it is the jth year and that B_j has been determined. Define v to be equal to j if $B_j \geqslant B_{j-1}$. If $B_j<B_{j-1}$ define v to be the unique layer such that:

$$\sum_{p=0}^{v} L_{pj-1} \geqslant B_j \geqslant \sum_{p=0}^{v-1} L_{pj-1} \tag{4}$$

In words, v will be the largest number (most recent layer) whose layer value is positive in year j. All layers above v will be zero, all layers below v will be unchanged from the previous year. The inventory layers can be updated using the following set of equations:

$$L_{kj}=L_{kj-1} \qquad \text{for } k<v \tag{5}$$

$$L_{kj}=B_j-\sum_{p=0}^{v-1} L_{pj-1} \qquad \text{for } k=v \tag{6}$$

$$L_{kj}=0 \qquad \text{for } k>v \tag{7}$$

As expected, where $v=j$ the new layer added by Eqn (6) is the difference between this year's and last year's adjusted inventory:

$$L_{jj}=B_j-B_{j-1} \tag{8}$$

Finally, let D_j be the dollar-value inventory figure to be reported in period j. Then:

$$D_j=\sum_{k=0}^{j} f_k L_{kj} \tag{9}$$

To express the change in dollar-value inventory during a period, use Eqn (9) for periods j and $j-1$ and the fact that L_{kj-1} equals zero for $k=j$, to calculate:

$$D_j-D_{j-1}=\sum_{k=0}^{j} f_k L_{kj}-\sum_{k=0}^{j} f_k L_{kj-1} \tag{10}$$

Then the use of Eqns (5)–(7) yields:

$$D_j-D_{j-1}=f_v L_{vj}-\sum_{k=v}^{j-1} f_k L_{kj-1} \tag{11}$$

$$=f_v B_j-f_v \sum_{k=0}^{v-1} L_{kj-1}-\sum_{k=v}^{j-1} f_k L_{kj-1} \tag{12}$$

From Eqn (6) it can be seen that the first and second terms of Eqn (12) represent the present year's value of the vth layer, L_{vj}, multiplied by the vth year's index. The third term subtracts from this amount the value of the previous year's LIFO layers which have been either changed or eliminated altogether. When $v=j$ the third term is zero while the second and third terms become the present dollar value of the layer being added, $f_j(B_j-B_{j-1})$.

By using gross margin, GM_j, for year j, changes to the income statement resulting from changes in purchases of inventory items can be demonstrated. Let P_j be the cost of items purchased during the period and R_j the revenue. Then:

$$GM_j=R_j-P_j+D_j-D_{j-1} \tag{13}$$

For convenience, the subscript j will be deleted from the variables GM, R and P for the remainder of the paper. Equation (12), with Eqn (2) to expand the value of B_j, then yields:

$$GM=R-P+f_v B_j-f_v \sum_{k=0}^{v-1} L_{kj-1}-\sum_{k=v}^{j-1} f_k L_{kj-1} \tag{14}$$

$$=R-P+(f_v/f_j)\sum_{i} c_{ij} q_{ij}$$

$$-f_v \sum_{k=0}^{v-1} L_{kj-1}-\sum_{k=v}^{j-1} f_k L_{kj-1} \tag{15}$$

Throughout this paper, various partial derivatives will be taken with respect to the amount of item i in inventory, q_{ij}. To do this precisely, the amount must be assumed to be a continuous variable (e.g. tons or gallons). For discrete inventory items the derivatives represent an estimate of the rate of change over the interval between two integer values.

Assume a situation in which a change in the inventory level of an item q_{ij} would have no impact on the calculation of the index value f_j. This would be the case, for instance, when an externally generated index is used. Then:

$$\frac{\partial GM}{\partial q_{ij}}=\frac{\partial R}{\partial q_{ij}}-\frac{\partial P}{\partial q_{ij}}+\frac{f_v}{f_j} c_{ij} \tag{16}$$

In the case of decisions regarding the replenishing of inventory, revenue is unchanged so that $\partial R/\partial q_{ij}=0$. In addition, the change in the total amount of purchases will be the cost of the item purchased or $\partial P/\partial q_{ij}=c_{ij}$. Thus:

$$\frac{\partial GM}{\partial q_{ij}}=-c_{ij}+\frac{f_v}{f_j}c_{ij}=c_{ij}(f_v/f_j-1) \qquad (17)$$

Equation (17) is consistent with the impact usually ascribed to the replacement of earlier years' LIFO layers. If inventory layers are being removed, then the impact on gross margin of replacing a unit of inventory will not be zero. During times of inflation, f_j is assumed to be greater than f_v. When a replacement unit is purchased, the gross margin will be decreased by an amount equal to $(1-f_v/f_j)c_{ij}$. Since this decrease in gross margin generates lower tax payments, managers who wish to conserve assets are often advised to increase inventory rather than eliminate any but the highest LIFO layers. Conversely, managers who, for any number of reasons, desire to show higher net income will find it advantageous to break into lower and lower layers. For the case where $v=j$ (i.e. no previous year's layer is eliminated) $f_v/f_j=1$, and gross margin is unaffected by changes in physical inventory. Historical inflation virtually assures that, on average, the cost index will increase over time. The ratio f_v/f_j becomes increasingly smaller, and the impact of changes in inventory at lower layers grows. When a single-item inventory pool is used, $f_v=c_v$ and $f_j=c_j$, and Eqn (17) indicates that a purchase of inventory changes the gross margin calculation by an amount equal to c_v-c_j. This is the usual result for the single-item model.

The analysis of this section is based on an externally generated index. The next section provides more discussion of internally generated inventory cost indices.

THE INVENTORY COST INDEX

There are three types of approach to the development of inventory cost indices. Two of these, the link-chain and the double-extended methods, are developed from the firm's own cost experience. The third approach is to use a published price index which applies to the type of inventory being valued. Indexing in the latter fashion is often difficult, since the data tend to be drawn from a set of products that can be very different from the inventory pool being valued and are only provided after the period in question. Forecasting for planning purposes and understanding the results using a published price index can also be very difficult. Consequently, most large firms tend to choose one of the first two methods. An exception is found in the retail LIFO method, where the IRS forces the use of certain externally generated indices for tax purposes.

For the double-extended method the index is equal to the ratio of the total value of a sample of the existing inventory in terms of the current costs to the total value of the sample inventory in terms of the base year's costs. For the link-chain method, the index is calculated in a recursive manner by multiplying the last year's index by the ratio of the values of the existing inventory sample at current cost and last year's cost. Formally, let:

S = the set of inventory items sampled to form the cost index where $S\subset I$,
g_j = the inventory cost index for year j based on the double-extended method,
h_j = the inventory cost index for year j based on the link-chain method.

Then the formulas for the two indices are given by:

$$g_j=\sum_S c_{ij}q_{ij}\Big/\sum_S c_{i0}q_{ij} \qquad (18)$$

$$h_j=\left(\sum c_{ij}q_{ij}\Big/\sum c_{ij-1}q_{ij}\right)h_{j-1} \qquad (19)$$

It is possible that the choice of sample items may be different from one year to the next, which would require an index, j, on the sample set, S. This notation is not necessary here and will not be used, in the interest of simplicity. The value of both indices are defined to be one in the base year. In general, $h_j\neq g_j$ unless the physical inventory count of each item in the pool is the same for all years.

To determine gross margin as a function of one of the internally generated indices, replace f_j by g_j or h_j in Eqn (15). For the double-extended index and $v\neq j$:

$$GM=R-P+g_v\left(\sum_S c_{i0}q_{ij}\Big/\sum_S c_{ij}q_{ij}\right)\sum_I c_{ij}q_{ij}$$

$$-g_v\sum_{k=0}^{v-1}L_{kj-1}-\sum_{k=v}^{j-1}g_kL_{kj-1} \qquad (20)$$

For the case where an inventory layer is being added, $j=v$:

$$GM=R-P+\sum_{I} c_{ij}q_{ij} - \left(\sum_{S} c_{ij}q_{ij} \Big/ \sum_{S} c_{i0}q_{ij}\right) B_{j-1} \quad (21)$$

Similar equations can be derived for the link-chain index. Results for this method will be presented but not derived explicitly.

To determine the impact of purchases on gross margin, when layers are not being eliminated, differentiate Eqn (21) with respect to q_{ij}. For inventory items not in the set sampled to create the index (i.e. $i \in I-S$):

$$\frac{\partial GM}{\partial q_{ij}} = -\frac{\partial P}{\partial q_{ij}} + c_{ij} = 0 \quad (22)$$

For $i \in S$:

$$\frac{\partial GM}{\partial q_{ij}} = -\left[\left(c_{ij}\sum_{S} c_{i0}q_{ij} - c_{i0}\sum_{S} c_{ij}q_{ij}\right) \Big/ \left(\sum_{S} c_{i0}q_{ij}\right)^2\right] B_{j-1} \quad (23)$$

Equation (22) conforms to the usually stated hypothesis. As can be seen in Eqn (23), however, purchase of items contained in the sample set will not lead to the expected result. To explore this situation more fully use Eqn (18) to show:

$$\frac{\partial GM}{\partial q_{ij}} = -(c_{ij} - g_j c_{i0}) B_{j-1} \Big/ \sum_{S} c_{i0}q_{ij} \quad (24)$$

Now define r to be the fraction of total replacement value of inventory being sampled to create the index:

$$r = \sum_{S} c_{ij}q_{ij} \Big/ \sum_{I} c_{ij}q_{ij} \quad (25)$$

Then using Eqn (2) with g_j replacing f_j, and Eqn (18), it can be shown that:

$$\sum_{S} c_{i0}q_{ij} = rB_j \quad (26)$$

which yields:

$$\frac{\partial GM}{\partial q_{ij}} = -(c_{ij} - g_j c_{i0})(1/r) B_{j-1}/B_j \quad (27)$$

If the items purchased are in the index sample set, S, the purchase of inventory at a cost of c_{ij} will not affect gross margin only if the ratio of the present cost to the base year cost exactly equals the inventory cost index for this year, i.e. $g_j = c_{ij}/c_{i0}$. In the more likely situation, purchase of inventory can cause gross margin either to increase or decrease, depending on the sign and magnitude of $c_{ij} - g_j c_{i0}$. Let w_i be the item i index defined by the ratio c_{ij}/c_{i0}. Then rewrite Eqn (27) to express the change in gross margin resulting from a purchase of item i as:

$$c_{ij}(g_j/w_i - 1)(1/r) B_{j-1}/B_j \quad (28)$$

Purchases of items whose costs have increased rapidly, relative to the inventory cost index (i.e. $g_j/w_i < 1$), will cause a decrease in gross margin, while purchases of items that have shown resistance to inflation relative to the pool will lead to an increase in gross margin. This result is contrary to the usually accepted belief that additions to inventory when all earlier years' layers remain intact will not impact gross margin. Moreover, the change in gross margin can be either positive or negative, regardless of the direction of the overall cost index. It is common to have a significant percentage of inventory sampled for cost index purposes. The possibility of inadvertent changes in annual income figures caused by a large purchase of inventory at the end of the year is high, even in the case where no inventory layer has been eliminated.

The magnitude of the impact of purchases of inventory on the ending inventory calculation and thus gross margin can be significant. To give an idea of the range of this impact, divide Eqn (28) by c_{ij} to yield the change in gross margin per dollar spent on inventory:

$$(g_j/w_i - 1)(1/r) B_{j-1}/B_j \quad (29)$$

The value of g_j/w_i will always be positive and the value of r can range from zero to one. No guidelines regarding sample size exist other than prescribing the use of a statistically accurate technique. In practice, it seems to be reasonably common for r to range from 0.5 to 1. Very small values of r, however, are possible. For the case of inventory layers being added, a value of $B_{j-1}/B_j = 1$ would imply that the new layer, L_{jj}, was zero. As the added layer gets increasingly larger this ratio will decrease to zero. Since the largest impact occurs when the new layer is just beginning to be added, B_{j-1}/B_j will be set to one. Table 1 then shows the resulting change in gross margin per dollar spent to purchase inventory item, i, for various values of r and g_j/w_i.

Table 1. Gross Margin per Unit of Expenditure for Inventory Purchases when a layer is being Added

	Sample size, r					
g_j/w_i	0.10	0.40	0.50	0.60	0.80	1.00
0.10	−9.00	−2.25	−1.80	−1.50	−1.13	−0.90
0.25	−7.50	−1.88	−1.50	−1.25	−0.94	−0.75
0.50	−5.00	−1.25	−1.00	−0.83	−0.63	−0.50
0.75	−2.50	−0.63	−0.50	−0.42	−0.31	−0.25
1.00	0.00	0.00	0.00	0.00	0.00	0.00
1.50	5.00	1.25	1.00	0.83	0.63	0.50
2.00	10.00	2.50	2.00	1.67	1.25	1.00
3.00	20.00	5.00	4.00	3.33	2.50	2.00
4.00	30.00	7.50	6.00	5.00	3.75	3.00
5.00	40.00	10.00	8.00	6.67	5.00	4.00
6.00	50.00	12.50	10.00	8.33	6.25	5.00
7.00	60.00	15.00	12.00	10.00	7.50	6.00
8.00	70.00	17.50	14.00	11.67	8.75	7.00
9.00	80.00	20.00	16.00	13.33	10.00	8.00
10.00	90.00	22.50	18.00	15.00	11.25	9.00

A quick glance at Table 1 shows that a purchase of inventory items potentially can have a very large impact on gross margin and, through it, on taxes paid and reported net income. Ten per cent sample sizes and a g_j/w_i ratio of ten will cause purchases to have an effect on gross margin ninety times greater than dollars expended to purchase inventory. A ratio such as this is not impossible, considering that the base year for double-extended indices can be as old as fifty years (e.g. over fifty years the ratio of a 10% to a 5% growth rate will be greater than ten). With 50% sample sizes, purchases of items whose index is twice the rate of the pool's index (i.e. $g_j/w_i = 1/2$) will lead to decreases in gross margin of $1 for every dollar spent on inventory. With the same sample size, purchase of an item whose index is one-third of the pool's cost index would increase gross margin by $4 for every dollar spent. To appreciate the potential magnitude of the impact that purchase of inventory items can have on gross margin, take a company with a net income of $100 000 on sales of $1 million and an inventory value of $250 000. Using the last two examples above, an addition to inventory of $25 000 (10%) would cause an estimated 25% decrease or a 100% increase in net income before tax. The materiality of this result implies that inventory managers must select carefully both the amount and timing of inventory to be purchased.

Dollar-value LIFO allows very diverse pools of inventory items to be formed. Indeed, large pools are actively encouraged by accounting practitioners. It would not be unusual to find individual items whose cost changes over the time period LIFO had been in effect were quite different from the pool's inventory cost index. Table 2 shows the 1971 (base), 1988, and 1989 cost figures for eighteen items in a very large manufacturer's raw materials pool. The ratio g_j/w_i ranges, in this case, from 0.795 to 4.818, even though the costs for these particular raw materials are very interdependent. Inventory pools made up of thousands of manufacturing items have still wider variability from one items to the next.

Similar calculations can be made when using the link-chain method to generate the inventory cost index, h_j. In this case Eqn (29) becomes:

$$(h_j/w_i - 1)(1/r)B_{j-1}/B_j \qquad (30)$$

where w_i is defined to be $h_{j-1}c_{ij}/c_{ij-1}$. In this definition the value c_{ij-1}/h_{j-1} can be viewed as an estimate of c_{i0}. Because h_{j-1} is an average index value, variability in gross margin should be mitigated by use of the link-chain method. The double-extended method, however, is generally preferred in practice because it is believed to reflect more accurately the actual price changes of inventory over time. The IRS prefers this method to the point that companies using the link-chain method must show that use of the double-extended method was not possible or practical.

Table 2. Costs for Manufacturer's Raw Materials

Item	1971 Base cost	1987 cost	1988 cost	1987 w	1987 g/w	1988 w	1988 g/w
1	4.373	18.017	19.929	4.120	1.137	4.557	1.071
2	4.234	17.807	19.929	4.206	1.114	4.707	1.037
3	4.514	18.038	19.379	3.996	1.172	4.293	1.137
4	4.869	19.515	21.253	4.008	1.169	4.365	1.118
5	5.732	30.563	29.248	5.332	0.879	5.103	0.957
6	3.971	20.488	20.611	5.159	0.908	5.190	0.941
7	3.692	16.669	18.766	4.515	1.038	5.083	0.960
8	3.804	10.889	18.112	2.863	1.637	4.761	1.025
9	3.186	14.037	12.166	4.406	1.063	3.819	1.278
10	3.894	14.454	15.450	3.712	1.262	3.968	1.230
11	2.770	16.319	15.748	5.891	0.795	5.685	0.859
12	71.050	72.000[a]	72.000[a]	1.013	4.623	1.013	4.818
13	9.650	34.313	42.633	3.556	1.318	4.418	1.105

Inventory cost index (1987) 4.685.
Inventory cost index (1988) 4.882.
[a] Estimated.

In the case where lower LIFO layers have already been invaded, conventional wisdom suggests that during times of inflation, replacing inventory at lower layers will reduce gross margin and thus tax payments. The following calculations will show that this result will not necessarily occur for dollar-value LIFO. To demonstrate this possibility, take the partial derivative of gross margin with respect to q_{ij} using Eqn (20):

$$\frac{\partial GM}{\partial q_{ij}} = -\frac{\partial P}{\partial q_{ij}} + g_v \frac{\partial}{\partial q_{ij}}\left(\sum_I c_{ij}q_{ij}\sum_S c_{i0}q_{ij}\Big/\sum_S c_{ij}q_{ij}\right) \quad (31)$$

For $i \in I - S$:

$$\frac{\partial GM}{\partial q_{ij}} = -c_{ij} + c_{ij}g_v/g_j = c_{ij}(g_v/g_j - 1) \quad (32)$$

and for $i \in S$:

$$\frac{\partial GM}{\partial q_{ij}} = c_{ij}[-1 + g_v/g_j + g_v/(w_i r) - (g_v/rg_j)] \quad (33)$$

$$= c_{ij}(g_v/g_j - 1) + c_{ij}[(g_j/w_i - 1)(1/r)]g_v/g_j \quad (34)$$

Equation (32) is in the same form as when an external index is used, since in both cases the index calculation is not changed by the purchase of inventory items. Equation (34) highlights the situation when the item purchased is included in the sample index. The first term of this equation is the same as Eqn (32), while the second term is similar to that determined for the case where an inventory layer is added. Comparing Eqns (34) and (28) reveals this similarity. The change in gross margin can be either positive or negative as a result of an inventory purchase that replaces old inventory layers. To see this, divide Eqns (33) or (34) by c_{ij} to get the change in gross margin per dollar spent:

$$-1 + g_v/g_j + g_v/(w_i r) - g_v/(g_j r) \quad (35)$$

or

$$-1 + g_v/g_j[1 + (g_j/w_i - 1)(1/r)] \quad (36)$$

Since r is positive but less than 1, the sum of the second and fourth terms of Eqn (35) must be non-positive. Consequently, a sufficient condition for Eqn (35) to be negative is that the sum of the first and third terms be non-positive. This condition would require that g_v/w_i be less than r. If $g_v/w_i > r$, the condition can arise that Eqn (35) is positive (i.e. gross margin increases as a result of a purchase) even during times of inflation. With 100% sampling $g_v/w_i < 1$ is a necessary and sufficient condition for gross margin to decrease as a result of purchasing inventory.

Table 3 shows the magnitude of the change in gross margin per unit of expenditure as a function of g_j/w_i and g_v/w_i for $r = 0.1$, $r = 0.5$ and $r = 1$. Recall that g_j/w_i is the present ratio of the pool index to the item index and that g_v/w_i is the ratio of the vth year's index to the item index. Inspection of Table 3 demonstrates that changes in gross margin can be either positive or negative, and that they are always

Table 3. Change in Gross Margin per Unit of Expenditure for Inventory Purchases at Lower Levels

		g_v/w_i							
		0.10	0.25	0.50	1.00	2.00	3.00	4.00	5.00
Sample size, $r=0.10$									
g_j/w_i	0.10	−9.00	−21.00	−41.00	−81.00	−161.00	−241.00	−321.00	−401.00
	0.25	−3.60	−7.50	−14.00	−27.00	−53.00	−79.00	−105.00	−131.00
	0.50	−1.80	−3.00	−5.00	−9.00	−17.00	−25.00	−33.00	−41.00
	0.75	−1.20	−1.50	−2.00	−3.00	−5.00	−7.00	−9.00	−11.00
	1.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	2.00	−0.45	0.38	1.75	4.50	10.00	15.50	21.00	26.50
	4.00	−0.23	0.94	2.88	6.75	14.50	22.25	30.00	37.75
	6.00	−0.15	1.12	3.25	7.50	16.00	24.50	33.00	41.50
	8.00	−0.11	1.22	3.44	7.88	16.75	25.63	34.50	43.38
	10.00	−0.09	1.28	3.55	8.10	17.20	26.30	35.40	44.50
Sample size, $r=0.50$									
g_j/w_i	0.10	−1.80	−3.00	−5.00	−9.00	−17.00	−25.00	−33.00	−41.00
	0.25	−1.20	−1.50	−2.00	−3.00	−5.00	−7.00	−9.00	−11.00
	0.50	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
	0.75	−0.93	−0.83	−0.67	−0.33	0.33	1.00	1.67	2.33
	1.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	2.00	−0.85	−0.63	−0.25	0.50	2.00	3.50	5.00	6.50
	4.00	−0.83	−0.56	−0.13	0.75	2.50	4.25	6.00	7.75
	6.00	−0.82	−0.54	−0.08	0.83	2.67	4.50	6.33	8.17
	8.00	−0.81	−0.53	−0.06	0.88	2.75	4.63	6.50	8.38
	10.00	−0.81	−0.53	−0.05	0.90	2.80	4.70	6.60	8.50
Sample size, $r=1.00$									
g_j/w_i	0.10	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	0.25	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	0.50	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	0.75	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	1.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	2.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	4.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	6.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	8.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00
	10.00	−0.90	−0.75	−0.50	0.00	1.00	2.00	3.00	4.00

negative for an item whose ratio g_v/w_i is less than the sample size. The impact of purchases on gross margin is diminished as the sample size is increased, as was the situation in the earlier case where an inventory layer was added. In addition, Table 3 shows that the magnitude of the impact on gross margin can vary considerably more than would be the case for a single-item inventory.

Variability of gross margin becomes more complex as more purchases are made. The direction of movement can change signs several times, causing gross margin to fluctuate up and down until reaching the point where an inventory layer is added. These fluctuations arise from the fact that the sign on the derivative in Eqns (35) or (36) is a function of g_v. Since g_v can vary independently from one layer to the next, the gross margin function can follow a roller-coaster path. Once a layer is being added, the direction of gross margin stays the same, since it is determined at this point by g_j/w_i. As more purchases are made this ratio will approach one from either above or below, depending on its value at the point where $B_j=B_{j-1}$.

Using the link-chain method for generating the inventory cost index results in the following equations which are comparable to Eqns (35) and (36) above:

$$-1+h_v/h_j+h_v/(w_i r)-h_v/(h_j r) \tag{37}$$

$$-1+h_v/h_j[1+(h_j/w_i-1)(1/r)] \tag{38}$$

As before, w_i is defined to be $h_{j-1}c_{ij}/c_{ij-1}$. Again, a critical factor for determining the variability of gross margin is the ratio h_j/w_i. When using the link-chain method, the range of values of w_i is only a function of the present and previous year's changes in costs. As a result, it is likely not to be as variable as for the double-extended method.

The above discussion indicates that it is possible for gross margin to be either increased or decreased as the result of a purchase of inventory. This statement is true whether an inventory layer is added during this period or a 100% sample is used to create the inventory cost index. When there is less than 100% sampling, it is also possible for the purchase of inventory to cause the elimination of LIFO layers. To see the circumstances whereby this possibility arises, use Eqns (2) and (18) to yield:

$$B_j=\sum_I c_{ij}q_{ij}\sum_S c_{i0}q_{ij}\Big/\sum_S c_{ij}q_{ij} \qquad (39)$$

Following a procedure similar to that in the previous section:

$$\frac{\partial B_j}{\partial a_{ii}}=(c_{ij}/g_j)[1+(g_j/w_i-1)(1/r)] \qquad (40)$$

From this equation it can be seen that the change in the adjusted inventory figure, B_j, will be negative as a result of a purchase whenever $g_j/w_i<1-r$. The ratio g_j/w_i must be positive, so that a 100% sample size precludes the possibility of a negative effect on adjusted inventory.

Equation (40) holds whether or not an inventory layer has been added. Figures 1 and 2 summarize the impact of a purchase on adjusted inventory, B_j, and gross margin, GM, for all inventory layers. It is possible for both B_j and GM either to increase or decrease as purchases are made. In addition, they can move in the same direction or in opposite ones. Where 100% sampling is used to form the cost

	$g_j/w_i>1-r$	$g_j/w_i<1-r$
$g_j/w_i>1$	Gross margin increases Adjusted inventory increases	Not possible
$g_j/w_i<1$	Gross margin decreases Adjusted inventory increases	Gross margin decreases Adjusted inventory decreases

Figure 1. Effect of purchases on gross margin and inventory when a layer is added

	$g_j/w_i>1-r$	$g_j/w_i<1-r$
$g_v/g_j+g_v/(w_ir)-g_v/(g_jr)>1$	Gross margin increases Adjusted inventory increases	Not possible
$g_v/g_j+g_v/(w_ir)-g_v/g_jr<1$	Gross margin decreases Adjusted inventory increases	Gross margin decreases Adjusted inventory decreases

Figure 2. Effect of purchases on gross margin and inventory when layers are eliminated

index, B_j can only increase, so that no elimination of LIFO layers can occur as the result of a purchase. GM can increase or decrease in value in this case. As more and more purchases of item i are made, the term g_j/w_i will eventually exceed $1-r$. To see this, note that r increases as purchases are made so that $1-r$ approaches zero. Meanwhile g_j/w_i approaches one as the ith item becomes a larger portion of the sample index. Thus B_j can start out as a decreasing function of q_{ij} but will eventually begin to increase.

Use of the link-chain inventory method leads to the same conclusion regarding variability in the value of adjusted inventory as was found with gross margin. Using the link-chain index, with the link-chain definition of w_i, Eqn (40) becomes:

$$\frac{\partial B_j}{\partial q_{ij}} = (c_{ij}/h_j)[1+(h_j/w_i-1)1/r] \qquad (41$$

Once again, the link-chain method will lead to reduced variability in the parameter in question.

An important final consideration for dollar LIFO calculations is the sample size used to generate the cost index. The impact of the statistical variance of the sample mean or the use of biased estimators can cause large errors in the calculation of gross margin. The extent to which external indices do not represent the physical items or the distribution of those items in the pool can also have a large impact on these calculations. To examine this impact, consider Eqn (15) and assume that the only variable in the system is the value of the index. In this case, use of f_j, g_j or h_j is interchangeable; thus:

$$\frac{\partial GM}{\partial f_j} = -B_{j-1} \quad \text{for } v=j \qquad (42)$$

and

$$\frac{\partial GM}{\partial f_j} = -(f_v/f_j)B_j \quad \text{for } v<j \qquad (43)$$

To estimate the magnitude of the effect of an error in the index value, let $B_{j-1}=B_j$, as before, and substitute $Q_j=f_jB_j$. A rough estimate of the impact on gross margin would then be:

$$\Delta \text{ Gross Margin} = -(\% \text{ error in } f_j)\times(f_v/f_j)\times Q_j \qquad (44)$$

This equation indicates that the change in gross margin is in the opposite direction of the index error. The magnitude of this change is equal to the percentage of error in the index times the current replacement value of the inventory mitigated (usually) by the ratio f_j/f_v. The mitigating factor applies to each layer affected by the error. Gross margin, for many companies, can be a fraction of the current inventory's replacement cost. A 10% or even 25% error (purposeful or otherwise) in the index calculation might be virtually impossible for an audit or review team to spot, but could lead to a significant misstatement of the reported net income for a period.

SAMPLE CALCULATIONS

The following data will be used to demonstrate the calculation of dollar-value LIFO using the double-extended method:

Item, i	*Base cost (\$), c_{i0}*	*Present cost (\$), c_{i4}*	*Quantity, q_{ij}*
1	10	45	1000
2	10	16	2000
3	10	13	3000
4	10	21	6000

Sample set, $S=\{1, 2, 3\}$ Sample size, $r=0.4793$ Previous year's adjusted inventory value, B $=\$124\,800$

Layer, v	*Layer index, g_v*	*Layer inventory value (\$), L_v*
0	1.000	120 000.00
1	1.250	2 400.00
2	1.300	2 215.38
3	1.667	184.62

Using these data the dollar-value LIFO amount D_4, is calculated:

$$g_4 = \sum_S c_{i4}q_{i4} \Big/ \sum_S c_{i0}q_{ij} = 116000/60000$$

$$= 1.933333$$

$$Q_4 = \sum_I c_{i4}q_{i4} = \$242\,000$$

$$B_4 = 242000/1.933333 = \$125\,172.41$$

$$B_4 - B_3 = \$372.41$$

$$L_{04} = \$120\,000.00$$

$$L_{14} = 2400.00$$

$$L_{24} = 2215.38$$

$$L_{34} = 184.62$$

$$L_{44} = 372.41$$

and

$$D_4 = 120000 + 2400 \times 1.25 + 2215.38 \times 1.3$$
$$+ 184.62 \times 1.667 + 372.41 \times 1.933$$
$$= \$126\,908$$

As an example where inventory layers are removed in spite of the purchase of inventory, assume that 200 additional units of item one have been purchased at year end:

$$g_4 = 125000/62000 = 2.016129$$

$$Q_4 = \sum_i c_{i4} q_{i4} = \$251\,000$$

$$B_4 = 251000/2.016129 = \$124\,496.00$$

$$B_4 - B_3 = (\$304.00)$$

$$L_{04} = \$120\,000.00$$

$$L_{14} = 2400.00$$

$$L_{24} = 2096.00$$

$$L_{34} = 0.00$$

$$L_{44} = 0.00$$

and

$$D_4 = 120000 + 2400 \times 1.25 + 2096 \times 1.3$$
$$= \$125\,724.80$$

To show the effect of an error in the calculation of g_j, assume that g_4 has been understated by 10% in the last example to be 1.814516. Then:

$$B_4 = \$251\,000/1.8141516 = \$138\,328.89$$

$$B_4 - B_3 = \$13\,528.89$$

$$D_4 = \$150\,731$$

Reducing g_4 by 10% leads to an increase in D_4 (and gross margin) of \$25 006. Use of Eqn (44) shows the estimated change in gross margin to be 10% × \$251 000 = \$25 100. This error can have a real impact on cash flow through such items as taxes, management compensation, and product price adjustments tied to gross margin.

Figures 3 through 6 demonstrate the impact of purchases on adjusted inventory, B_j, and gross margin, GM. For this purpose, the end-of-year inventory for the item being considered is set to

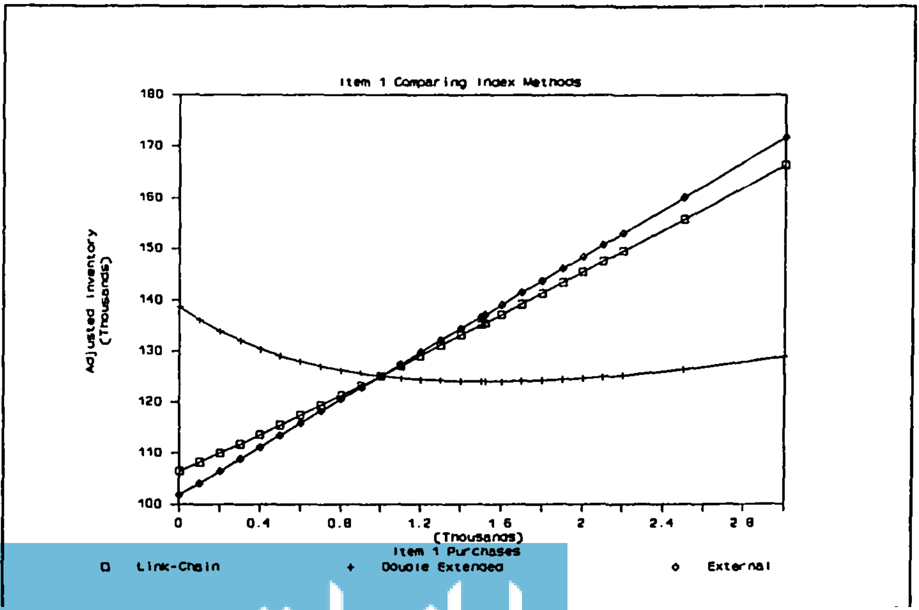

Figure 3. Purchases versus inventory.

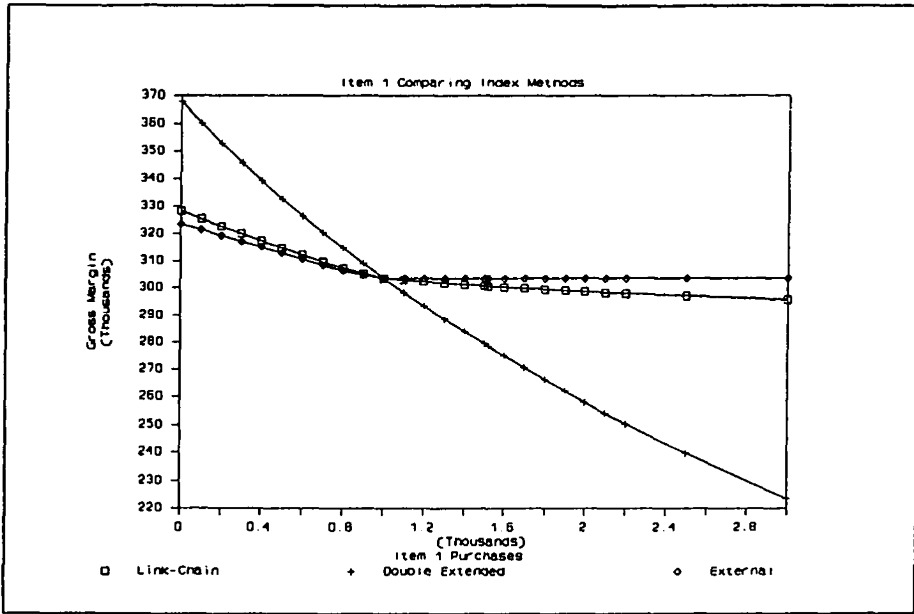

Figure 4. Purchases versus gross margin

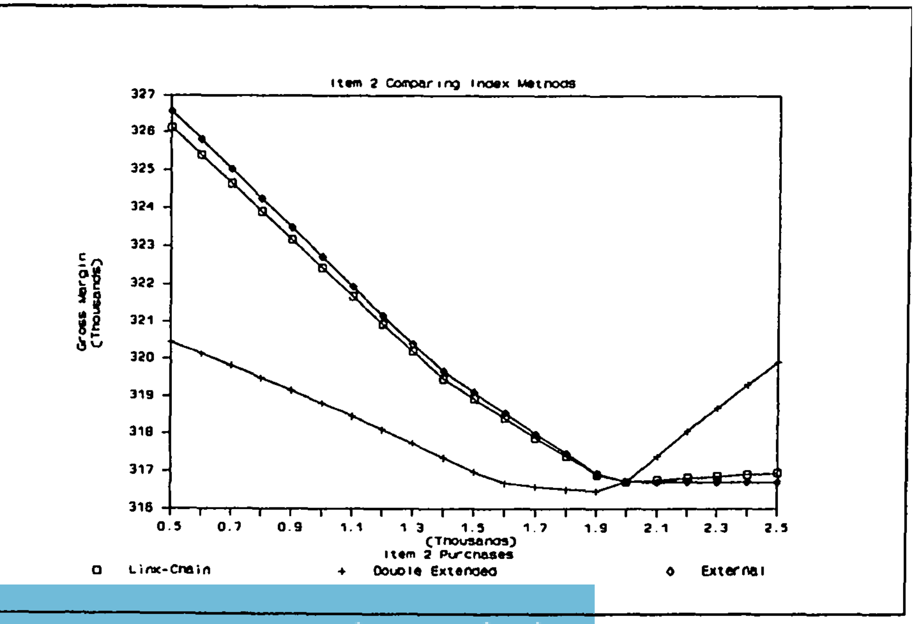

Figure 5. Purchases versus gross margin

zero initially. Each graph is the result of then allowing purchases to increase. Three different curves are presented in each figure. The first two curves are generated by using the double-extended and link-chain methods. All the figures are based on physical inventory remaining constant over the first three periods. Consequently, the two indices are equal for the first three periods, and will be equal at one point in the fourth. The index that occurs at this point ($g_j = h_j = 1.93333$) is used as a surrogate for an external index. This index is used to generate the third curve on Figs 3 through 6.

For item one using the double-extended method, the adjusted inventory is reduced for purchases up to slightly more than 1520 and then increases over the rest of the range (Fig. 3). When no units are purchased for item one, an inventory layer is added in the current period. As purchases increase, the adjusted inventory value is decreased until no layer is added: then layer three is eliminated, and finally a portion of layer two is eliminated. Inventory purchases of more than 1520 begin once again to add inventory layers. This example shows the unexpected situation in which inventory layers are added when inventory purchases are low and when they are high, but eliminated in between. Item one has a low value of g_j/w_t. As a result, g_j/w_t is less than $1-r$ over a portion of the range, thus causing adjusted inventory to move in this counter-intuitive fashion. Use of the link-chain method is much closer to the results achieved with the external surrogate.

Figures 4–6 show gross margin as a function of purchases for items one, two and three. Item one's cost has appreciated much more rapidly than the inventory cost index. Gross margin is reduced over the entire range as more and more purchases are made (Fig. 4). For the double-extended method, the magnitude of this decrease is considerably more than expected. In addition, gross margin continues to decrease even after a layer is added to inventory for the period. Item two demonstrates the situation when g_j/w_t is closer to one (Fig. 5). As inventory layers are added or eliminated, the rate of change of gross margin changes fairly drastically due to this item's rather sharp historical price changes. The graph for item three shows the unexpected result that gross margin increases in value as inventory purchases are made over the entire range of possible purchases (Fig. 6). This increase continues even in the area where a layer is being added to

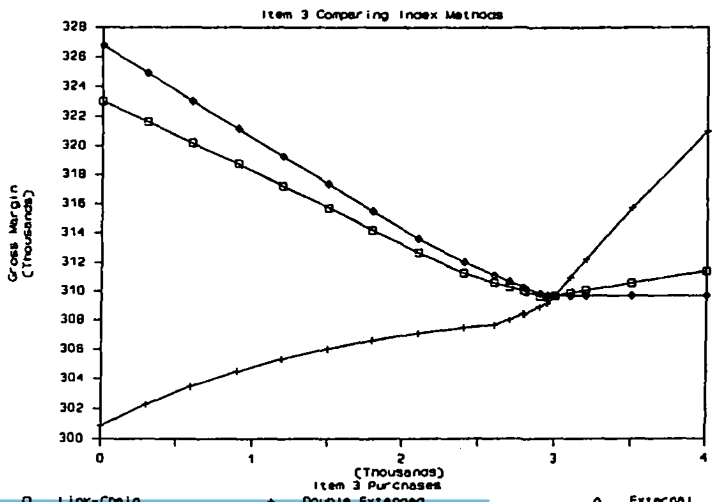

Figure 6. Purchases versus gross margin

inventory. For all three items, the impact of purchases is moderated by the use of the link-chain method.

CONCLUSIONS

Several results have been presented in this paper that are counter to conventional wisdom about LIFO inventories. Year-end additions to inventory can cause gross margin either to increase or decrease regardless of inflation or deflation. Independent of inflation levels, acquisition of inventory can cause layers to be eliminated when less than 100% samples are used to form index values. The criterion that determines the direction of the impact of a purchase on gross margin or adjusted inventory is the ratio of the overall cost index to the change in cost of the specific item being purchased. The magnitude of the impact of purchases on gross margin and adjusted inventory is a function of the size of the above ratio, the sample size, the present and previous year's adjusted inventory value, and the historic values of the index being applied. Use of the link-chain method seems to moderate the impact of the index calculation on both gross margin and adjusted inventory.

Dollar-value LIFO is the most widely used LIFO inventory method. When this methods is employed both the index type and sample size must be considered. On the surface, externally generated indices eliminate the impact of the sample index on the resulting inventory level, gross margin, and taxes. These index values do not reflect the actual inventory in question. As a result, the index is incorrect relative to the specific inventory. Consequently, their use virtually guarantees that important balance sheet and income statement values will be incorrectly stated. If an internally generated index is used, the sample size must be determined. The potential impact of inventory purchases from items in the sample set can be very large if sample sizes are small. Relatively small purchases of specific inventory items will have a large impact on gross margin, cash flow and reported inventory. In addition, small sample sets would lead to a potentially large statistical error in the index value and a still larger impact on reported inventory and gross margin. Large sample sizes would mitigate the error effect and the magnitude of the impact of purchases, but would not eliminate them altogether. In addition, these sample sizes would increase the possibility that the purchase of an item would directly affect the index calculation.

The results found in this paper have implications in several arenas. From the point of view of the prudent manager, relative cost changes for the individual items must be considered when evaluating potential additions to inventory at year end. The management of gross margin cannot be viewed as tantamount to controlling the addition or elimination of inventory layers. Care must be taken if financial statement values are used as input into managerial decision making. From the auditor's point of view, index sampling practices should be carefully scrutinized from a broader perspective than the statistical validity of the index. Large year-end purchases should be reviewed in more depth when the individual item's cost changes do not closely mirror the pool's index value. Special attention should be spent on potential misrepresentation of the sample index. From the point of view of the tax planner, consideration should be given to the adoption of different indices for tax and financial purposes. From the point of view of the IRS, the policy of insistence on the use of the double-extended method for cost indices should be carefully reviewed. From the point of view of empirical accounting researchers, the impact of external index surrogates on empirical studies should be studied to determine whether these approximations obfuscate the potential results. In addition, hypotheses regarding manager's selection of cost-flow methods based on their ability to control income should be tested. On the one hand, managers might shy away from LIFO because of the complex and unpredictable results. On the other, the level of control on gross margin that can be achieved through end-of-year purchases might give managers a desirable planning tool. Finally, from the point of view of analytical accounting researchers, models must be specifically constructed to include the impact of dollar-value LIFO on optimal decision making.

Acknowledgements

The author would like to acknowledge the assistance of the members of the Accounting and Finance Workshop at the University of Kansas. In addition, he is grateful for the help of Bill Patterson and Kirk Putman of Arthur Andersen and Company, Chuck Warner and John Jensen of Ernst and Young, Lynn Markel, Dan Kelly, Brad Hall and Dick Knudson of Koch Industries, Ed Cockerell of Tension Envelope Company, Robert Lavery of the Securities and Exchange Commission and Ray Lauver of the Financial Accounting Standards Board. This work was partially supported by The University of Kansas General Research Allocation # 3926.

REFERENCES

American Institute of Certified Public Accountant (1984). *Identification and discussion of certain financial accounting and reporting issues concerning LIFO inventories*, AICPA, November.

G. C. Biddle (1980). Accounting methods and management decisions: the case of inventory costing and inventory policy. *Journal of Accounting Research* **18**, Supplement, 235–80.

G. C. Biddle and F. W. Lindahl (1982). Stock price reaction to LIFO adoptions: the association between excess returns and LIFO tax savings. *Journal of Accounting Research* **20**, Autumn, 551–88.

G. C. Biddle and R. K. Martin (1985). Inflation, taxes, and optimal inventory policies. *Journal of Accounting Research* **23**, Spring, 57–83.

G. C. Biddle and R. K. Martin (1986). A stochastic inventory model incorporating intra-year purchases and accounting tax incentives. *Management Science* **32**, June, 714–30.

R. M. Bowen and G. M. Pfeiffer (1989). The year-end LIFO purchase decision: the case of Farmer Brothers Company. *The Accounting Review* **64**, January, 152–171.

J. K. Butters (1949). *Inventory Accounting and Policies*, Vol. 1: *Effects of Taxation*, New York: Riverside Press.

L. G. Chasteen, R. Flaherty and M. O'Connor (1987). *Intermediate Accounting*, New York: Random House.

M. A. Cohen and R. Halperin (1980). Optimal inventory order policy for a firm using the LIFO inventory costing method. *Journal of Accounting Research* **18**, Autumn, 375–89.

W. R. Cron and R. B. Hayes (1989). The dollar value LIFO pooling decision: the conventional wisdom is too general. *Accounting Horizons* **3**, No. 4, December, 57–70.

H. Z. Davis, N. Kahn and E. Rozen (1984). LIFO inventory liquidations: an empirical study. *Journal of Accounting and Research* **22**, Autumn, 480–96.

N. Dopuch and M. Pincus (1988). Evidence on the choice of inventory accounting methods: LIFO versus FIFO. *Journal of Accounting and Research* **26**, Spring, 28–59.

D. E. Kieso and J. Weygandt (1986). *Intermediate Accounting*, New York: John Wiley.

J. M. Reeve and K. Stanga (1987). The LIFO pooling decision: some empirical results from accounting practice. *Accounting Horizons* June, 25–33.